A unified method of identification and optimization of airfoils for aircrafts, turbine and compressor blades

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Abstract

Topics below are rather undesired, but important, outcome not yet completed research on the aircraft airfoils, turbine and compressor blades, parametric design of airfoils, establishing the relationships based on the results of experiments in a wind tunnel, developing databases for determining the relationships between airfoil parameters and lift and drag coefficients. Reliable database created as a result of the research work allows to simulate the wind tunnel. Very early on, however, it was necessary to extend the developed specialized software for a new applications, and it meant the need for generalization of software, e.g. for gas turbine engines, propellers, etc. But after some time it turned out, that in order to achieve the required accuracy, the changes are needed in the underlying assumptions, set decades ago. In addition, coordinate measuring machines and systems, and associated software were not always as accurate as expected. Concepts how to solve it and develop software carrying out these tasks are presented in the article. It is like to withdraw from the old path and look for a new path that will lead to the reliable data base. Processes related to air or gas flow should be similarly defined in all the specialized software applications (e.g. aircrafts and turbine engines). Accuracy ($10^{-9}$ mm) achieved in virtual measurements within the integrated system can be used to verify the results of CMM and other measuring systems, provided that an appropriate software has been developed.

KEYWORDS: airfoils, design optimization, combinatorial-cyclic method, turbine engines
1. Introduction

Over the last two decades, passenger and military aircrafts, became a fully high technology products, and design, engineering, manufacturing of these aircrafts is carried out entirely within integrated CAD/CAM/CAE systems. It should be emphasized that in these systems, all the tasks related to the definition of the geometric shape of the structure are realized by Nurbs geometry concepts. Because this geometry can define all surfaces, and solids that occur in engineering practice, therefore, the software-based identification and optimization of even the most complex shapes have become feasible, at least, by a virtual prototyping approach. There is a meaningful difference, as far as time-consuming factor is concerned, whether an interactive mode is used or program-oriented definition of any engineering problem, described by using a build-in basic system language (e.g. GRIP in Siemens NX system, updated APT, etc.). Such a programming is becoming the necessary qualification for engineers of high technology products, particularly, when the highest quality, optimization, CNC manufacturing, and reverse engineering are of a primary concern.

The main role in the developed software, needed to solve given tasks, plays the combinatorial-cyclic method of optimization. The structure of subprograms in the method is fit to different area of applications, since one subprogram is treated as target-oriented and defining the objective function as well as selecting either a deterministic model or artificial intelligence model. There has been gathered a considerable experience in the following main areas of applications:

(1) Analysis of all errors in manufacturing processes, even on the most complex surfaces (e.g. airplanes, turbines, propellers, car bodies, etc.), moreover, on each stage of CAD/CAM technology; eliminating the necessity of a precise set-up of the part being measured on a CMM table or for scanning system; verifying the accuracy of CMM itself and applied software. The specialized software is of a great importance for effective calculation of dimensional deviations between the virtual product model and the part machined, defined by a cloud of points (even more than 2 million). In assembled physical objects the total error can be divided on errors of surfaces machined and assembling errors.

(2) Design optimization, according to required accuracy of location and orientation of principal axes of inertia or other design objective functions, e.g. minimizing the deviation between principal axis and axis of rotation; optimization of parametric design, where parameters are optimization variables; it enables us to effectively apply reverse engineering techniques (max. number of optimization variables=15). Virtual balancing, as a variety of design optimization, has become a necessity in virtual prototyping of gas turbine engines.

(3) Establishing the optimal mathematical formula for interrelationships among variables, based on experimental data or parametric design data; it works as an extension to multiple regression analysis not requiring the linearity. Usually, we can consider additionally 2nd or 3rd degree, power or exponential equations.
(4) Solving the equation systems, linear and non-linear, particularly, where the number of unknowns is less than number of equations; mostly applied to analytically defined surfaces (e.g. a sphere, plane, cylinder, cone, ellipsoid, etc.). The method identifies these surfaces and solid bodies with high accuracy from a cloud of points, obtained from coordinate measuring systems. Measuring the reference sphere (30, 50, or 100 points), we can effectively determine the current accuracy of the CMM.

To achieve higher geometric accuracy of complex shapes, airfoil shapes included, and overall increase in surface quality it is necessary to take full advantage of capabilities available in integrated CAD/CAM/CAE systems as well as in a new generation of CNC production equipment and coordinate measuring machines. It should be remembered that one of the main advantages of Nurbs geometry is that we can easily calculate the shortest distances between points and solid body models, and more importantly, give them signs '+' or '-', what means –outside- or -inside- of the body. But in this context, if we want to accurately measure the airfoil we have to replace it with the solid body model, even if it is virtual measurement.

Rules for the airfoil analysis and experimental research in aerodynamics were established decades ago, when computer systems, based on Nurbs geometry, were not tools of everyday use for engineers, and therefore, these rules should be revised. These revised rules must provide two main features that could not be guaranteed in the existing computational procedures and practical steps of implementation.

The two main features are:

- building an accurate CAD airfoil model from the point set, wherein, high accuracy is achieved by the optimization of curves that make up the model surfaces, and then analyzing the CAD model errors by using combinatorial-cyclic method of optimization;
- the ability to accurately verify the geometric errors of the physical model, manufactured by CNC machine tools according to the CAD model, and then measured by coordinate measuring systems.

When the CAD / CAM / CAE system is extended by specialized programs ensuring these two features, then even the geometry of the aircraft, directly from the manufacture or repair, can be geometrically identified by a coordinate measuring. Combinatorial-cyclic method of optimization [1,2] provides computation of the differences in position and orientation of the two systems, the coordinate system of the CAD model and the measurement coordinate system, which, as yet, had been replaced by the time-consuming and geometrically inaccurate set-up of the object being measured. It should be noted that the accuracy achieved with such an approach in the virtual prototyping allows to determine permanent deformations after extreme testing in flight.

To achieve these new features the addition of a new specialized software for almost all the major modules of the system, is required.
Fig. 1 shows the developed specialized software in different modules of the system to meet the challenges of identifying and optimizing processes in various stages of CAD, CAM, and CAE. The new software is presented after sign ‘+’. This software is also generalized in nature and allows by the same definitions to describe computational processes for aircrafts and turbine engines. Of course, the range of applications can be relatively easily extended to other similar products, especially, to those where a reverse engineering and accuracy of complex shapes plays an important role, e.g. airscrews, ships and yachts, cars, satellite dishes, etc.

2. A revised method of airfoil definitions

When we want to increase the accuracy of the airfoil fitting to the given set of points and also to extend definitions on turbine blades, compressor blades, airscrews, and ship propellers we need to make changes to the previously applied rules in an airfoil design process, airfoil manufacture process, and coordinate measurement of the airfoil shape machined. A new approach for defining the airfoils from the set of points has already been presented in [2], but in practice, the verification and the need to adapt the airfoils for the turbines and compressors, required significant modification. The contents of current description, though apparently similar, differs in details and in the sequence of steps.

Points, underlying the accurate calculations of airfoils, are derived from the published tables of standardized airfoils (NACA), from airfoils tested in wind tunnels, and, more and more often, from digitization or scanning of real objects, photographs or images on the internet, from available technical documentations or specifications on the internet. Assuming that in this approach coordinate errors of points are inevitable, absolutely the first step must be to reduce to the maximum impact of these errors.
The key to accurately determine the location, orientation and length of the chord and camber line is a calculation of two extreme segments of the given airfoil, i.e. the leading edge and trailing edge segments. The leading edge segment is always the curve of 2nd degree, mostly circles, but ellipses, parabolas, and hyperbolas are also possible. The trailing edge segment is a line or the curve of 2nd degree as in the previous edge. The main assumption is that, at first, we start with circles only and compute the first segment of camber line, which also determines the axis of symmetry. Then we go to circles and ellipses, to compute the first segment of the camber line, which is a straight line. The standard deviation gives an indication what is the best fit, a circle or ellipse. It is worth to notice that in a definition of an ellipse, a circle is the specific case only. The procedure can be extended to parabolas and hyperbolas.

As the results of this initial step, we get:

- a point that marks the beginning of a new X-axis, the beginning of a new chord, the beginning of the first segment of a camber line; this point is so calculated that determines the first segment of the camber line and also the axis of symmetry of the 2nd degree curve, which was not previously possible for ellipses, parabolas, and hyperbolas;
- a point that marks the end of a new X axis, the end of a new chord, the end of the last segment of a camber line;
- two points of tangency on the leading edge and two points of tangency on the trailing edge (or two intersection points).

If the new X axis is not the same as in the coordinate system of the points, the set of points must be transformed by rotation and translation. So far, there were no such transformations, and errors arising from that, even more than 0.5 mm or 0.5 degree, could not be determined by the coordinate measurement. Now, by applying the method presented in this paper, it is no longer impossible.

After the corrected coordinate system has been established, upper and lower segments of the airfoil are optimized to fit to the transformed set of points. To optimize the airfoil as a whole (four segments), the combinatorial-cyclic method of optimization is necessary.

It is worth noting that all the airfoils consist of four different segments, each of these segments should be a separate curve with clearly defined beginning and end of the segment. Each of the four segments is determined from a given set of points, and the number of points can be very different - tens, hundreds, or even thousands.

There are well-defined geometric constraints for particular segments of the profile, resulting from research in aerodynamics and gas dynamics, and determining the approximate number of points required to define the optimal curve for each segment. The key to accurately determine the location, orientation and length of the chord is a calculation of two extreme segments of the given profile, i.e. the leading edge and trailing edge segments. The leading edge segment is always the curve of 2nd degree, mostly circles, but ellipses, parabolas, and hyperbolas are also possible. The trailing edge segment is a line or two lines or the curve of 2nd degree as in the leading edge.
To generalize algorithms for all airfoils, either airplane or turbine and compressor blades, in both segments the starting, ending, and midpoints are precisely calculated. Therefore, the chord must be defined as the straight line joining the midpoint of the leading edge and the midpoint of the trailing edge, and these points may not always be the extreme points of the profile. It is worth repeating that if the starting point of the chord or the angular position of the chord are not exactly in line with the chord established from the set of points, e.g. joining the extreme points, then you need to transform the existing set of points to the coordinate system of the new chord.

The airfoil as the optimal curve can be presented in the Nurbs geometry definition:

\[
nurbs1=bcurve/fit, p(1..n), toler, t, degree, d
\]

where:
- \( p \), \( t \), \( d \) - obtained from the optimization process,
- \( p \) - selected points (e.g., 15 to 25 points),
- \( t \) - max. distance between the selected points and curve,
- \( d \) - degree of the upper and lower segments of the airfoil (mostly 3). The method used to optimize the profile curve can be extended to solid bodies of an airplane wing, stabilizer, and fuselage. Also, turbine blades, compressor blades, etc. are optimized by this method, but, of course, the software for an automatic generation of geometric models must be developed.

The optimal airfoil with the mean chord length (e.g., 100 mm, 1000 mm or any other) are then parameterized. The main aim of parameterization is establishing the optimal mathematical formula for interrelationships between characteristics of the airfoil and parameters, e.g., \( c_l \) or \( c_d \) as a function of \( p(1..n) \). It is possible, to read parameters from the airfoil and, inversely, to build the airfoil from parameters. The airfoils can be divided into classes, depending on the specific properties of airfoils or standard length of the chord. Usually, for increasing the accuracy of the calculation of aerodynamic characteristics, e.g., \( c_l \), \( c_d \) for all airfoils resulting from scaling the basic airfoil.

Of course, the complete and reliable database enables to develop the software simulating a wind tunnel. Instead of manufacturing a physical model of airfoil shape for a wind tunnel, it is possible to predict \( c_l \) and \( c_d \) from airfoil parameters. The detailed procedure has been presented in the paper [2], but after have gathered some databases, it turned out that some modifications and extensions are required.

Successes of CFD software encourage to a new approach to aerodynamic research, particularly, to problems hitherto unsolved. The unified method for all airfoils, i.e., wings, stabilizers, longitudinal section of an aircraft fuselages, turbine blades, compressor blades, airscrews, etc. makes it easy to extend the application of wind tunnel research. Before we start the wind tunnel research, we have to know accuracy of the designed CAD model and accuracy of the CNC machined physical model.
Fig. 2. Accurate calculation of leading and trailing edges of the wing airfoil and of the turbine blade airfoil. As a result, we may adjust the length and position of the chord, what enables to transform points defining the airfoil. Then the airfoil as a Nurbs curve can be optimized in relation to the transformed set of points. The middle point and two boundary points in edge curves or edge lines are of great importance in the presented concept. The figure has been generated by the optimization program from a cloud of points (hundreds of points). Airplane thin airfoils and compressor airfoils are similar to left and right airfoils, respectively, except that the edge radius can be as small as 0.5 mm. Heretofore, because of the large camber angles on the leading edge of the turbine and compressor blades, the same method of defining the edges, as for airplane airfoils, could not be used.

General conic: CIRCLE(0,1),ELLIPSE(1),PARABOLA(3),HYPERBOLA(2)
A*x**2 + B*x*y + C*y**2 + D*x + E*y + F = 0
n1= 8, n2= 5, n3= 1 n4=15, n5=12
Conic index (as above): ehp=1, first point p( 8)
ellipse center: 149.7212, 26.0472, .0000
semimajor: 150.0000
semiminor: 149.8732
tilt angle: 10.0000
start angle: 211.3381
end angle: 542.8950
start point:  19.8343, -51.8256, .0000
end point:  -2.0562, 18.4729, .0000
standard deviation for all points p( 8..12): sigma= 0.1312
OPTIMUM GCONIC-CURVE:p(8),p(5),p(1),p(15),p(12);
Remove all points below and insert the middle, and two boundary points
p(8)=POINT/-2.0562, 18.4729, .0000
p(5)=POINT/ 5.8911, 73.4382, .0000
p(1)=POINT/ 46.8690, 135.1525, .0000
p(15)=POINT/ 57.6299, -92.7142, .0000
p(12)=POINT/ 19.8343, -51.8256, .0000

Fig. 3. Printout from the program showing the outline of algorithms for the general conic optimization, i.e. for both edges of the airfoil. The middle point and two boundary points (tangency points) are also computed in the program.

Points for design and points from CMM measurement are the base for computing sigma, as the error measure. It is especially important when the geometric CAD model has not been defined in the same system as the CNC program. A generalized geometric identification of all airfoils in use, is presented graphically on the example of an airplane airfoil and the airfoil of a turbine blade, fig.2. A thin airfoils of airplanes and airfoils of compressors actually differ only in leading edge radii and thickness in relation to two airfoils presented, but the leading and trailing edges have a significant impact on the chord. A geometrically accurate airfoil, optimized in relation to a set of points, is an essential condition for the transition to solve other tasks, carried out by the dedicated software.

It is recognized that almost always when we get an airfoil as a cloud of points, we have no certain information about the precision with which the X-axis of measurement system coincides with the airfoil chord. To indicate the validity of the problem
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of airfoil accuracy, the most important aspects have already been presented in the Introduction above. Circles, ellipses, parabolas or hyperbolas are computed from 15 to 30 points taken from the area around both edges.

The results of iterative computations with the ever decreasing number of points are as shown in the printout after each run with a new number of points from e.g. 30 to 5. Therefore, to minimum 5, because the curves of second degree are defined as general conics, fig. 3. This is actually the first step in identifying the leading edge and, in particular, to determine the angle of the first segment of camber.

Similarly, it can also be shown the optimization of the airfoil as one NURBS curve, linking together three segments, i.e. the segment of leading edge with upper and lower segments. The results of combinatorial-cyclic method are printed in paper [2]. Airfoil geometry has been optimized as CAD solid body model. Precise identification of the airfoil geometry means that the geometrical model requires only the development of a tool path for CNC milling machines with 3- or 5-axis controlled. The presented sequencing gives a very large guarantee, that we can achieve a high accuracy and repeatability, assuming that we proceed within the integrated system.

3. Accuracy of CAD models and CNC machined parts, and developed CAQ extensions for assemblies

CAD model surfaces shaped for air or gas flow around, could not be measured, so far, that it was clear that each point of the measurement is above or below the concerned surface. Introducing the combinatorial-cyclic method to the CAQ module allows it. Analysis of errors is carried out both for the CAD model and physical object machined.

Even the surfaces of analytic geometry as spheres, general cones, slanted cylinders can be used to test the accuracy of the hardware and software of CMM used.

In the paper [2] we presented the wide range of goals, which could not be fully achieved, because of errors in an early stage of the research. Parametric design is necessary for an optimization but even optimized model cannot be manufactured by any standardized machining processes. Some CAD models for wind tunnel tests (e.g. airplane fuselage, wings, blades) should be, if possible, machined according to the direction of air flow. Experiences from these approach necessitated adjustments in specialized software, and it is introduced below.

This way of presenting the results is a standard for the combinatorial-cyclic method of optimization, which was applied to the analysis of coordinate measurement of F-16 airplane model, designed for research in a supersonic wind tunnel. There is not known any other method that would result in an accurate calculation of the surface errors from hundreds, or even hundreds of thousands, of points from coordinate measurement (CMM) of the physical object (3D). Surface errors, measured at a given points, are obtained after the CAD model (which is the base for the CNC program) has been positioned inside the set of points, so that the sum of squared distances from all points of measurement to the surface reached minimum.
Effective visualization of errors on the measured surface is only possible because each rotation and translation of the CAD model in relation to the points changes visible points and its colors. Of course, the colors represent error values into and out of the material. Reading the text with detailed documentation takes more time, but, in general, it is essential in practice.

Previously, it was possible to only measure the distance between the characteristic points of the design, which could be a large number, but it is not comparable to what is currently giving digitizing or scanning the surface of the design. For example, to identify the sphere meant coordinates of the center and the radius of the sphere. For this, it was enough 4 to 10 CMM points, but when these points was 100 or more, we had no effective mathematical tools to determine the surface errors. It must be remembered that the arithmetic mean is not a good criterion, but the accepted criterion is the minimum sum of squares and resulting from that $\sigma$.

It should be emphasized that the combinatorial-cyclic method eliminated the need to set the object to be measured in accordance with the coordinate system of the CAD model, which accounted for over 50% of the time-consuming process of measurement, and the measurement involved only, a dozen or more, specific points or sections of the design. The method can also be used when the object as a whole must be measured with different reference points, i.e. with different origins of the coordinate system for the measurement. The same problems are also present in these technical objects like airliners (Boeing, Airbus), gliders, helicopters (Sikorsky), car body dies, turbine and compressor blades, yacht hulls, hip replacements, reflectors and satellite dishes, etc. For these objects, this method has been used and modified since 1998.
Attention must be drawn that not only the outer surface of the airplane has a lot of surfaces composing the whole, which are machined separately and then assembled. Car body outer surface is composed of, at least, 11 surfaces, which are pressed by different dies, and dies, generally, do not produce sheetmetal with the same accuracy. If we want to achieve a drag coefficient $c_d<0.30$, we need to test the model of the car in the wind tunnel, and before that check the accuracy of the model. Very often this requires that the complex coordinate measurements are performed on work surfaces of dies to accurately determine the shape errors. Determining the differences between the sheet metal surfaces and working surfaces of dies allows to compute springback errors, as indeed it was the first field applications of combinatorial-cyclic method.

Before we use the geometric CAD model in the CAM module, in order to prepare CNC machining program, first we should also analyze CAD model errors, except that the base of comparison is then the same points that were used to define the model. Scanning errors or other size errors can be corrected. The final inspection of CAD solid body model is the mass analysis, where we check if model is not encumbered by a serious mass distribution errors, which could then be transferred to programs for CNC milling machines. The most important is whether the center of mass is in the plane of symmetry of the solid body model and whether the two principal axes of inertia are in the plane of symmetry, and whether one of these axes, at least with the permissible error, coincides with the principal axis along a fuselage in right-hand coordinate system.

Coordinate measurements were made in one setup with possibility to group points: (1) F-16 model as a whole, (2) the center and rear fuselage, (3) the front fuselage with the cockpit and nozzle inlet, (4) wings as a whole, right and left, (5) the right wing, (6) the left wing. First and foremost is the error analysis of the model as a whole, and it means that we sum up the machining errors and assembling errors of all components. It was not until the next steps we submit further analysis of surfaces (2,3,4,5,6) that make up the model. This increases the time-consuming research, but also the results of these analyzes are crucial to assess the reasons for the resulting errors, and allows the separation of the surface errors from the assembling errors of parts that make up the multi part object, which is the airplane model.

In the technical metrology a measure of error is the standard deviation $\sigma$ and $3 \times \sigma$. Since the central and rear fuselage are the one part of the model to which are mounted the front fuselage and wings, so it can be assumed that the errors $\sigma$ and $3 \times \sigma$ of this part are 100%, and in relation to this part are measured translations and rotations of all other parts, which are assembling errors.

In the case of a milling the part in one setup on the CNC milling machine, $\sigma$ is a measure of the surface errors arising from the quality and accuracy of the machine, CNC program quality, but so far these errors were measured in selected sections of the part machined. In current approach the part machined is measured and then compared with the CAD model throughout its volume. Errors resulting from failure to accurate setup of the part in relation to the coordinate system of the model also affect the assembling errors.
Of course, each integrated CAD / CAM / CAE system accomplishes the modeling and manufacturing within the CAD and CAM modules and never redefines the geometric model, from which the CNC program is developed. After any change of the model all optimization analyses must be repeated.

The analysis takes into account errors that the model is composed of many parts machined separately and comparative analysis of these parts is given in the table as a sigma / sigma fuselage (2). The short comparative analysis of parts within the airplane assembly model has been shown in Tab.1.

In order to fully understand all of the errors in the CNC machining and in the assembling the model, it is necessary to present an analysis of the errors in the various parts that make up the model. It only indicates the cause of errors.

3.1. Assembling errors in relation to fuselage (2)

(1) F-16 model as a whole
\[ \begin{align*}
\text{dx} &= 101.1733 - x(2) = -0.1833 \text{mm} \\
\text{dy} &= -188.2687 - y(2) = 0.0022 \text{mm} \\
\text{dz} &= -1.9083 - z(2) = -0.0863 \text{mm}
\end{align*} \]

Summary of the results of error analysis

<table>
<thead>
<tr>
<th>Model surface errors</th>
<th>% of points sigma</th>
<th>% of points +/- sigma</th>
<th>% of points 3*sigma</th>
<th>% of points +/- 3*sigma</th>
<th>sigma fuselage(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) F16 model</td>
<td>0.1276mm</td>
<td>81.42%</td>
<td>0.3828mm</td>
<td>98.17%</td>
<td>365.61%</td>
</tr>
<tr>
<td>(2) Fuselage central</td>
<td>0.0349mm</td>
<td>77.35%</td>
<td>0.1046mm</td>
<td>99.94%</td>
<td>100.00%</td>
</tr>
<tr>
<td>(3) Fuselage front</td>
<td>0.1423mm</td>
<td>88.41%</td>
<td>0.4270mm</td>
<td>97.98%</td>
<td>407.74%</td>
</tr>
<tr>
<td>(4) Wings as a whole</td>
<td>0.1107mm</td>
<td>73.00%</td>
<td>0.3322mm</td>
<td>99.32%</td>
<td>317.19%</td>
</tr>
<tr>
<td>(5) Wing right</td>
<td>0.0408mm</td>
<td>66.51%</td>
<td>0.1224mm</td>
<td>99.42%</td>
<td>116.91%</td>
</tr>
<tr>
<td>(6) Wing left</td>
<td>0.0547mm</td>
<td>96.42%</td>
<td>0.1641mm</td>
<td>99.87%</td>
<td>156.73%</td>
</tr>
</tbody>
</table>

Fuselage central(2):
\[ \begin{align*}
\text{x(2)} &= 101.3566 \text{mm} \\
\text{y(2)} &= -188.2709 \text{mm} \\
\text{z(2)} &= -1.8220 \text{mm}
\end{align*} \]

\[ \begin{align*}
\text{dalfa_x} &= 0.6358 - \text{alfa_x}(2) = -0.0094 \text{ degree} \\
\text{dbeta_y} &= -3.3172 - \text{beta_y}(2) = -0.0682 \text{ degree} \\
\text{dgama_z} &= -0.0575 - \text{gama_z}(2) = 0.0169 \text{ degree}
\end{align*} \]

Tab. 1. The analysis of F-16 model errors
(4) Two wings as a whole
\[ dx = 99.8758 - x(2) = -1.4808 \text{mm} \]
\[ dy = -188.7938 - y(2) = -0.5229 \text{mm} \]
\[ dz = -1.9488 - z(2) = -0.1268 \text{mm} \]
\[ d\alpha_x = 0.6968 - \alpha_x(2) = 0.0516 \text{ degree} \]
\[ d\beta_y = -3.4490 - \beta_y(2) = -0.1900 \text{ degree} \]
\[ d\gamma_z = -0.2243 - \gamma_z(2) = -0.1499 \text{ degree} \]

Asymmetry error of two wings dy = -0.5229 mm, requires a thorough error analysis for the two wings separately. Error indicates that the two wings are moved along the Y-axis dy = -0.5229 mm from the XZ symmetry plane; the problem is the first, but not the only, indicating that the two wings as a whole does not meet the symmetry condition in relation to the symmetry plane of the airplane model.

(5) Right wing
\[ dx = 100.8611 - x(2) = -0.4955 \text{mm} \]
\[ dy = -187.0223 - y(2) = -1.2486 \text{mm} \]
\[ dz = -1.3273 - z(2) = 0.4947 \text{mm} \]
\[ d\alpha_x = 1.0144 - \alpha_x(2) = 0.3692 \text{ degree} \]
\[ d\beta_y = -3.3280 - \beta_y(2) = -0.0690 \text{ degree} \]
\[ d\gamma_z = 0.5747 - \gamma_z(2) = 0.6491 \text{degree} \]

(6) Left wing
\[ dx = 102.7037 - x(2) = 1.3471 \text{mm} \]
\[ dy = -190.8077 - y(2) = -2.5368 \text{mm} \]
\[ dz = -1.5773 - z(2) = 0.2473 \text{mm} \]
\[ d\alpha_x = 0.5148 - \alpha_x(2) = -0.1304 \text{ degree} \]
\[ d\beta_y = -3.2849 - \beta_y(2) = -0.0259 \text{ degree} \]
\[ d\gamma_z = 0.0034 - \gamma_z(2) = 0.0778 \text{ degree} \]

### 3.2 Closing remarks on computed results

Validation and verification of the method is carried out as follows:
- around 1000 points are measured on the CAD model surface using system tools, the same as in the virtual coordinate measurement;
- the points are subjected to transformation in the space;
- the applied combinatorial-cyclic method identifies the transformation, i.e. translation and rotation around the axis of the model system, and the resulting standard deviation \( \sigma = 0 \), namely 0.000000, i.e. \( 10^{-9} \), since this is the accuracy of Siemens NX7.5 system.

Rules applied to the analysis of CMM measurements of airplane models can also be used to other multi-part objects (assemblies) in full scale. Of course, the full scale of the object often means that we cannot use coordinate measuring machines (CMM) but laser measuring systems with larger measurement errors.

Verification of results of CMM measurement by the virtual measurement, as described above, confirmed the results for all parts from which has been assembled the model.
Results of the analysis can be verified by virtual measurements for all parts of the model, including the model as a whole, but the results are the same, $\sigma = 0$. Therefore, the front fuselage was chosen, because in this part the CNC machining errors are maximum. The last part of this analysis has been shown below:

**OUTSIDE AND INSIDE ERRORS**

Ten maximum outside distances:

- $\text{ptm}(84)$: $x = 148.7551$, $y = 42.0552$, $z = -4.6323$, $\text{dist} = 0.000000001$
- $\text{ptm}(82)$: $x = 122.7822$, $y = 45.2660$, $z = -3.5096$, $\text{dist} = 0.000000001$
- $\text{ptm}(77)$: $x = 158.8327$, $y = -24.9500$, $z = -6.1918$, $\text{dist} = 0.000000001$

Ten minimum inside distances:

- $\text{ptm}(98)$: $x = 140.1554$, $y = 6.5764$, $z = 39.1135$, $\text{dist} = 0.000000000$
- $\text{ptm}(97)$: $x = 157.5730$, $y = 7.4581$, $z = 40.2845$, $\text{dist} = 0.000000000$
- $\text{ptm}(92)$: $x = 251.3148$, $y = 12.9132$, $z = 5.9603$, $\text{dist} = 0.000000000$

*Tab. 2. Excerpt from the printout; - dist-means an error.*

Since the use of software requires to know a running time for the computing task, below are given approximate times, if prerequisites are met.

**Prerequisites:**

- the computer at least 4 GB of RAM;
- the complexity as of the airplane model consisting of more than one component, machined separately;
- the number of points from coordinate measurements around 5000

When the required accuracy: 0.1 mm,  
the computation time from 15 min. to 1 hr. depending on initial values of iteration.

When the required accuracy: 0.01 mm,  
the computation time ca. 1.5 hr

When the required accuracy: 0.001 mm,  
the computation time ca. 3 hr

When the required accuracy: 0.0001 mm,  
the computation time ca. 4.5 hr

In mobile workstations of such class as Dell Precision M4700, you can, at the same time, calculate the model as a whole, and three or four other components, which significantly speeds up the execution of calculations for all components of the measured object.
Conclusions

In recent years, almost all achievements in aircraft technology are connected with the development of a new class of specialized software. This software is built mostly in languages relevant to a particular engineering system, specific to the industry. These languages are derived from Fortran and C++, but for engineering applications are much more efficient. Still, the software is never 100% reliable and requires constant testing and modifications. Thus, in the present study was paid so much attention to the results of the verification and testing. After all, even in the most modern aircraft we often encounter failures, resulting from the operation of the software.

References
